On the distance chromatic number of Hamming graphs

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Abstract

The distance power $G^{(d)}$ of a graph G has the same vertex set as G. Distinct vertices in $G^{(d)}$ are adjacent, if their distance in G is at most d. The distance chromatic number $\chi^{(d)}(G)$ of G relative to distance d is the chromatic number of $G^{(d)}$. For positive integers q, n the Hamming graph $H_{q,n}$ has as its vertex set the n-fold cartesian product $\mathbb{Z}_q \times \ldots \times \mathbb{Z}_q$, $\mathbb{Z}_q = \{0, 1, \ldots, q-1\}$. Vertices in $H_{q,n}$ are adjacent, if they differ in exactly one coordinate. We derive explicit formulas for the clique number $\omega(H_{q,n}^{(d)})$ and we determine some exact values of $\chi^{(d)}(H_{q,n}) = \chi(H_{q,n}^{(d)})$. For fixed d and n we show

$$\chi^{(d)}(H_{q,n}) = q^d + O(q^{d-1}).$$